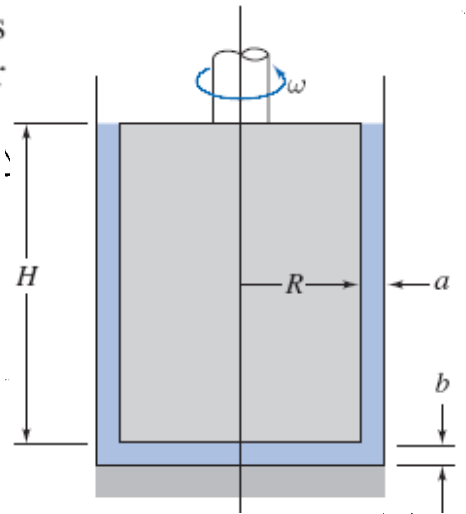


Problem 2.72

[Difficulty: 5]

2.72 A concentric-cylinder viscometer is shown. Viscous torque is produced by the annular gap around the inner cylinder. Additional viscous torque is produced by the flat bottom of the inner cylinder as it rotates above the flat bottom of the stationary outer cylinder. Obtain an algebraic expression for the viscous torque due to flow in the annular gap of width a . Obtain an algebraic expression for the viscous torque due to flow in the bottom clearance gap of height b . Prepare a plot showing the ratio, b/a , required to hold the bottom torque to 1 percent or less of the annulus torque, versus the other geometric variables. What are the design implications? What modifications to the design can you recommend?



Solution: Basic equation $\tau_{yz} = \mu \frac{du}{dy}$
 Assumptions: (1) linear velocity profile, (2) Newtonian liquid

(a) in annular gap

$\tau = \mu \frac{du}{dr} = \mu \frac{\Delta u}{\Delta r} = \mu \frac{U}{a} = \mu \frac{\omega R}{a}$
 $\text{Torque} = RF_r = R \tau A = R \mu \frac{\omega R}{a} (2\pi R H) = \frac{2\pi \mu \omega R^3 H}{a}$ (a)

(b) in bottom gap

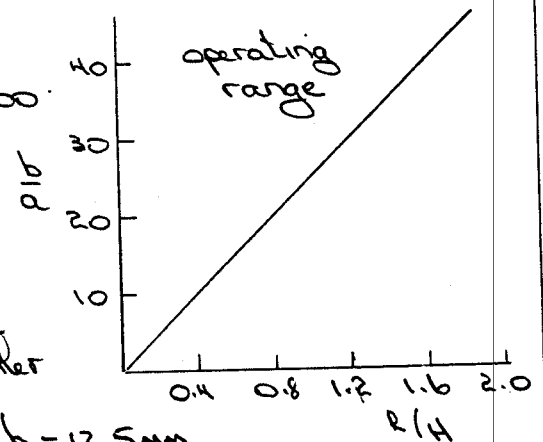
$\tau = \mu \frac{du}{dz} = \mu \frac{\Delta u}{\Delta z} = \mu \frac{U}{b} = \mu \frac{\omega r}{b}$ (varies with r)

$\text{Torque} = \int dT = \int r dF = \int r \tau dA = \int_0^R r \mu \frac{\omega r}{b} 2\pi r dr$
 $\text{Torque} = \frac{2\pi \mu \omega}{b} \int_0^R r^3 dr = \frac{2\pi \mu \omega}{b} \left[\frac{r^4}{4} \right]_0^R = \frac{\pi \mu \omega}{2b} R^4$ (b)

(c) For $T_{\text{bottom}} / T_{\text{annulus}} \leq \frac{1}{100}$, then

$\frac{T_{\text{bot}}}{T_{\text{an}}} = \frac{\pi \mu \omega}{2b} R^4 \times \frac{a}{2\pi \mu \omega R^3 H} \leq \frac{1}{100}$

$\frac{aR}{4bH} \leq \frac{1}{100}$
 or $\frac{b}{a} \geq 25 \frac{R}{H}$



(d) The plot shows the operating range. Specific design would depend on other constraints.

For $a = 1 \text{ mm}$ with $R/H = 1/2$ gives $b = 12.5 \text{ mm}$

(e) For a given value of R/H , the dimension b could be effectively increased by "hollowing out" the inner cylinder as shown by the dashed lines in the diagram above.